The Conceptual Understanding and Attitude towards Algebra at Secondary School Level

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Abstract

Understanding of the any concept is baseline for the attitude towards that concept. Conceptual understanding of algebraic concepts promoted positive attitude towards algebra. Positive attitude leads towards success in learning. This study was conducted to find out secondary school students' conceptual understanding and attitude towards algebra as well as the relationship between conceptual understanding and attitude towards algebra. 500 students enrolled at secondary schools of Islamabad were selected as sample of the study. An attitude scale and a test based on Structural Communication Grid were used as research instruments. For data analysis percentages and Pearson's Product Moment Correlation Coefficient was applied. Findings indicated that although 95% students have conceptual understanding of writing algebraic sentences but 50% students are unable to write logically correct open sentences. Almost 91% students have conceptual understanding of writing equations but 28-37% have no conceptual understanding of writing in-equation, linear and radical equations by using "<, >, ≤, ≥" symbols. Because of the gap in conceptual understanding of students weak, positive non significant relationship found between conceptual understanding and attitude towards algebra. Creative ways of teaching and practical activites are recommended to teachers for conceptual understanding of students about algebaic concepts.

Key Words: Conceptual Understanding, Attitude, Algebraic Equations, Structural Communication Grid, Secondary Students, Algebraic Sentences

Introduction

Conceptual understanding is knowledge that involves a thorough understanding of underlying and foundational concepts behind the algorithms performed in mathematics (Hope, 2006). Thus, it is cognitive mapping and involves a situation where students are able to understand and recreate the formulas and proofs without rote learning. Moreover, students are not bound to follow predefined ways, they are allowed to make own choices and apply their understanding for problem solving (Boaler, 2000). Conceptual understanding referred to understanding of mathematical ideas and procedures, and includes the knowledge of basic arithmetic facts ... [the ability to] identify and

apply principles, know and apply facts and definitions, and compare and contrast related concepts (Morehouse, 2007).

Attitude is a psychological construct associated with the cognitive, affective, and behavior of human learning (Dogbey, 2010). Attitude is unchangeable belief acquired due to the experience. It is linked with self-efficacy and self-concept. Yee (2010) defines attitude as an emotional disposition. It is based on four factors: enjoyment, value, motivation and self-confidence. Middleton, Ricks, Wright and Grant (2013) define attitude as intrinsic motivation, self-confidence, enjoyment and values. Attitude is built or developed based on above four factors. Student attitude towards mathematics is cited as contributing factor to their success in the subject (Mata, Monterio & Peixoto, 2012). Attitude towards mathematics depends on the understanding of the mathematical concepts. How they experience learning of mathematical concepts. What value they assigned to those mathematics concepts? Teachers use different methods and techniques of teaching to clear the concept for enhancing students understanding. In daily life understanding is the key to attitude. If one understand they exhibit positive attitude toward it (Goodykoonts, 2008). Clear understanding helps students in applying learned knowledge to solve the problems that motivate them which resulted in positive attitude (Rao, 2003).

Mathematics in general perceived as most difficult subject in school curriculum (Eshun, 2004 & Eshun-Famiyeh, 2005). This general perception is reflected in student performance over the years (Akpalu, Adaboh & Boateng, 2018). Among the challenges faced by the educational institutions are effective delivery and assessment of mathematics. To overcome these challenges a variety of teaching and assessment techniques have been advocated for the use in mathematics classroom. The problem not only lies in classroom instructions; students have negative perception about the nature of mathematics too which can be enhanced if teachers do not focus on the understanding of struggling students. There is a very sharp line between the perceived and conceived nature of mathematics, with little effort of teachers difficult perception about mathematics can change the attitude of students. Attitude is feeling and reaction to the context/situation; because of abstract nature of mathematics students possess negative attitude but the proper instructions and use of concrete material can improve student conceptual understanding (Steedly et. al., 2008). Students understanding has direct link with students attitude towards mathematics. The more they have understanding of the concept more they possess positive attitude. Algebra is a branch of mathematics considered as difficult as other branches. This study has focused on cognitive and affective aspects of students learning to investigate students' conceptual understanding and attitude towards algebra.

There are three objectives of the study:

1. To investigate the conceptual understanding of students about algebraic concepts by the use of Structural Communication Grid
2. To assess the attitude of students towards algebra
3. To find out the relationship between students conceptual understanding and attitude towards algebra

Literature Review

Mathematics is widely used in various fields and cover wide range of activities but along with it mathematics is not liked much which is evident by the decline in mathematics achievement (Akpalu, Adaboh & Boateng, 2018, Amponsah, 2010, Freduak-warteng 2005). The reason of decline is negative perception about mathematics. It seems difficult because of the gaps in

understanding of mathematical concepts which effects conceptual understanding. The conceptual understanding provides the base for attitude development. The earlier theorist of attitude describe attitude as "intellectual and neural state of willingness that is organized through experiences and gives direction to the individual behaviors towards something or some situation" (Bordous & Horowits, 2013).

The conceptual understanding is a process which is identified as: factual and procedural knowledge, connections, transfer and metacognition on the basis of all attitude is developed. It is basically a reaction to an action. According to sociological perspective attitude is verbal expression as an intention to act whereas according to psychological perspective it is verbal expression as behavior. Pitafi and Farooq (2012) stated attitude as a "psychological construct or latent variable, inferred from observable responses to stimuli which are assumed to mediate consistency and coherence among those responses".

Algebra is a branch of mathematics which gives holistic picture of mathematical concepts. It was introduced to secondary school students with the background they are mature enough to understand it. Because of its holistic nature this perception is reconstructed and recommended algebra may be introduced at elementary level (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005). It has twofold effects, it open prospects in educational as well as business opportunities. On the other side it increases the learning difficulties of students. The students' difficulties are because of poor understanding. It is teacher who bridges the gap and facilitates them to learn algebraic reasoning (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005).

Algebra is combination of symbols variables and constants. There is need to understand in order to recognize the properties of the algebraic equations, expressions and so on. Students understanding problems regarding algebra is also evident by Sengul and Erdogan (2014). They elaborated that students have less understanding of concept of equations, symbols and variables. The reason is that each concept is a building block for previous one so it is essential to clear the basic concepts for the advancement in learning.

The algebraic sentences and expression are basic concepts for students at secondary level. The algebraic expression like 2 + 5, 4a + 6, 3a + 6b, x + 3y -4z are related with any of the symbols like $<, >, \leq, \geq, \neq$ or $=$ etc to make an algebric sentence. The use of these symbols depends on students understanding. If they don't have conceptual understanding and conceptual clarity of these symbols they may fail. Basically the conceptual understanding is related to the mind and cognitive mapping (Drijvers, 2011).

Egodawatte (2011) highlighted the common mistakes and misconception of studentts in algebra. The findings indicated that most of the incorrect responses orginated from the misconceptions or misunderstandings of the concepts of algebra. Mostly students committed errors in algebric expression because of not having conceptual understanding or treat them according to the percieved patterns and acepted rules. They further have no clarity for the use of equality sign.

Linsell et. al. (2012) designed teaching approaches to help students of grade 9^{th} and 10^{th} for the development of their understanding of algebra. They gave suggestions to the teachers that students' achievemnet can be increased by focusing on the organized perspectives e.g. building a tool box of knowledge and skill of knowledge that students can use in their daily life.

Studeis also suggested innovative methods of assessment of the assessemnt of learning. Structural Communication Grid (SCG) is one of them it can be used for assessment as well as for diagnosis (Johnstone, Bahar & Hansell, 2000). Basically SCG is developed by students responses. Teachers asks question to write the correct answer and then breaks the answer in to component parts and scatterd randomly across the boxes of the grid. The students have to select the appropriate

boxes and arrange in to logical sequence. The size of the grid based on the age and stage of students. Johnstone, Bahar & Hansell (2000) suggested 3×3, nine boxes grid for secondary level and 4×3 or 3×4, twelve boxes for college level studnets. The content of the boxes can be words phrases, pictures, equations, definitions, numbers and formula's. As the content of the box can vary so it can be made suitable for visual and verbal thinkers. It completely eliminated the problem of guessing. The analysis of the result indicated the ways student knowledge is interlinked. The wrong seqence further used for diagnostic purpose. The misunderstandings and misconceptions can be handeled accordingly. The following SCG is used for this research.

A	B	C
D	E	F
G	H	I

Figure 1: Basic Structure of Structural Communication Grid

(Johnstone, Bahar & Hansell, 2000).

Methodology

The population of the study was secondary school students enrolled at Islamabad Secondary Schools. Islamabad Secondary Schools are public sector schools working under the administration of Federal Directorate of Education. The National Curriculum of Mathematics for Secondary Classes approved by National Curriculum Counsel is implemented through text books. The algebraic sentences and equation portion was selected from the books recommended by Federal Board of Intermediate and Secondary Education (FBISE). FBISE is responsible to conduct secondary school examination for Islamabad schools. Total students enrolled were 30003 in which 15109 female students and 14894 were male students (FDE, 2013). Stratified cluster sampling technique was used. 500 students 250 females and 250 male were selected as sample which is appropriate and representative according to Gay (2011).

Two instruments were used to conduct the study i.e. attitude scale for attitude measurement and Structural Communication Grid (SCG) for the conceptual understanding assessment about algebric concepts. The self developed Atitude Scale was consisted upon 20 items. These items were further assessed by using 5 point likert scale; rating from strongly agreed (SA) to strongly disagreed (SDA). Structural Communication Grid was a test used to find out the conceptual understanding about Algebra. SCG was based on 9 boxes i.e. 3×3. SCG was developed for two questions. The first question has 5 sub questions and second question has 4 sub questions. The students were free to select as many boxes as they want for the right answer of the given questions.

Both the instruments were validated by different experts having Mathematics, Psychology and Education background. The language and the sequence of the concepts were appropriate minor suggestions were received and questionnaires were updated. Pilot testing was conducted before

final administration of the instruments. The instruments were piloted in two secondary schools one male and one female. 50 students other than sample attempted the questionnaire and do not found any difficulty regarding language and understanding of the questions and statements. Before finalizing the attitude scale reliability coefficient was calculated on the pilot test data. The calculated value of r by Cronbach's alpha was 0.843 which is acceptable for social sciences research instruments. The reliability of test (SCG) was calculated by test-retest method. The value of r for test was 0.762. The test was administered before taking the students consent and explaining the purpose to create anxiety free environment. The second test was administered after one week. Data were collected from selected students by personal visits.

Results and Interpretation

Two instruments were administered to collect data regarding conceptual understanding of algebra and attitude towards mathematics.

Exploration of the conceptual understanding in Algebra

Test about the conceptual understanding of algebra contains two questions. Question one has four parts based on 'algebraic sentence', 'true sentence', 'false sentence' and 'open sentence' and second question has five parts i.e. 'Equation', 'In- Equation', 'Linear Equation', 'Radical Equation' and 'Simple Linear Equation'. Both the questions were developed on SCG containing 9 boxes i.e. 3×3 and measure conceptual understanding of sample students. The detail of students responses are presented in table 1 & 2.

Table 1:

Exploration of the conceptual understanding about Algebraic Sentences

Serial No	Questions	Frequency correct	% correct	Frequency incorrect	% incorrect
Q 1.1	Algebraic sentence	475	95%	25	5%
Q 1.2	True sentence	314	62.8%	186	37.2%
Q 1.3	False sentence	284	56.8%	216	43.2%
Q 1.4	Open sentence	250	50%	250	50%

%=percentage

Above table shows students responses on conceptual understanding about different concepts of algebra. Students have showed divers responses about understanding of algebraic sentences. 95 % students understand the algebraic sentence but students' response on other concepts is alarming. Almost 40-50% students are incorrect on true sentence, false and open sentence; which showed they have no understanding of these concepts.

Table 2:

Exploration of the conceptual understanding about Equations

Serial No	Questions	Frequency correct	% correct	Frequency incorrect	% incorrect
Q 2.1	Equation	458	91.6%	42	8.4%
Q 2.2	In-Equation	358	71.6%	142	28.4%
Q 2.3	Linear Equation	316	63.2%	184	36.8%
Q 2.4	Radical Equation	316	63.2%	184	36.8%
Q 2.5	Simple linear equation	344	68.8%	156	31.2%

%=percentage

The question on equations seems easy to students as 91% students are correct on this question. But 30-40% student's responses are incorrect on in-equation, linear equation, radical equation and simple linear equations. These questions appears difficult to students

The analysis of students responses showed about half to one third sample have problem in understanding the concepts of algebra. This situation needs teachers and school administrator's attention.

Table 3:
Relationship between students' conceptual understanding and attitude

Variables	N	Pearson 'r'	p- value
Attitude towards Algebra Conceptual Understanding in Algebra	500	0.043	0.333

The value of correlation coefficient (.043) and corresponding p-value (.333) shows that there is a positive weak but non- significant relationship between students' conceptual understanding in algebra and their attitude towards algebra.

Findings

Conceptual understanding of students was measured by test and attitude towards mathematics by attitude scale. Test was based on two questions containing four and five parts. Students were supposed to respond on structural communication grid. It was 3×3 boxes. The boxes have some numbers and symbols. Students have to take the number, expression or symbol from the box and write algebraic sentences, true sentences, false sentences and open sentences in question 1. Same was for second question. Student various response on questions regarding algebraic sentences and equations were received. It seems writing algebraic sentence is easy for students as 95% students were correct on this part of test. But 40-50% student's response on other types of algebraic sentences was incorrect. There may be two reasons, one is writing algebraic sentence needs only some symbols like "$<, >, \leq, \geq, \neq$ or $=$" which don't need any logic where as writing open sentence, true and false sentence it need some logic to be correct. The same response noted against first part of second question. The correct response on this part is 91%. It has logic that students recognize equality sign it looks easy to them select number or expression from grid and connect with equality sign. But 28-37% studnets feel difficult to select correct symbol to write logically correct in-equation, linear equation, radical equation or simple linear equation.

In first question part four 50% students were incorrect on writing open sentences. Open sentences further has two types: equation and in-equation which were the parts of second question on test. When students responses on those parts were analysed it showed different result only 9 % students were incorrect on writing equation and 28% were incorrect on writing in-equation. It indicated these students has difficulty in using only "$<, >, \leq, \geq$" symbols because they have sucessfully used "$\neq$ or $=$" symbols for writing correct equation and in-equation.

The second indicator was attitude towards algebra. Attitude as defined by Yee (2010) and Middleton, Ricks, Wright & Grant (2013) is based on four factors enjoyment, self-confidence Value and motivation. The value of Pearson's "r" was .043 and p = .333 which indicated non significant weak realtionship between conceptual understanding and attitude towarsds algebra. This weak relation has logic as students have low or no conceptual understanding of the use of "$<, >, \leq, \geq$" symbols in test. It resulted in low motivation, low self-confidence, they are not enjoying so they do not value it ultimately it affects their attitude towards algebra.

Discussions

The findings of the study showed almost one third of sample has no understanding of the concepts of algebra. They are incorrect on given questions. The reason is that it seems difficult to them. This thought is evident from previous research conducted by different researchers (Akpalu, Adaboh & Boateng, 2018; Amponsah, 2010; Mensah et. al. 2005; Freduakwarteng, 2005). They found students have problem in conceptual understanding because of two reasons one they think mathematics is difficult subject so the algebra too and second is they found gaps in understanding of the building blocks of the concepts of algebra. The first is related to attitude called affective domain and second is related to conceptual understanding called cognitive domain of Bloom's taxonomy of educational objectives. The findings further support the studies conducted by Egodawatte (2011), Sengul and Erdogan (2014) and Jennison and Beswick (2010) they found understanding of the concept is the key to develop positive attitude in students.

Although the relationship is not significant but it gives a direction if someone has clarity of concept he/she must have positive attitude towards it. So the main thing is concept clarity which needs teacher's attention. Once the student has conceptual understanding they have positive attitude towards it and once they have positive attitude they will start enjoying it their self-confidences increase they will be motivated to get more and more from studies. It acts as a ladder once students stand on it; they move further and reach to destination. As per above discussion the conceptual understanding and attitude both works together and leads to each other. During teaching; teachers' responsibilities are twofold. They must be sure that if students have conceptual clarity and they possess positive attitude which help them to get good grades and success in life. Linsell et. al. (2012) designed teaching approaches to help teachers for the development of understanding of algebra which can enhance the positive attitude in students. But if students' have negative attitude towards the subject no doubt how hard they act they will not be succeeded in their life.

Recommendations

1. As findings indicates that 50% students did not have the conceptual understanding of an open sentence, 43.2% students did not have conceptual understanding of a false sentence, 37.2% did not have the conceptual understanding of a true sentence, 36.8% did not have the conceptual understanding of a radical equation, 31.2% students did not have the conceptual understanding of a simple linear equation. Therefore there may be some interesting methods of teaching to these algebraic concepts that students can easily understand it. For example, teachers may use algebraic puzzles: it would enhance their conceptual understanding. Continues encouragement for students should be there by their teachers.
2. The finding shows that there was a positive weak but non-significant relationship between students' conceptual understanding in algebra and their attitude towards algebra. Therefore, more time and practical activities may be given to students for developing their conceptual understanding in the algebra, in this way students can take interest in it. Teachers may adopt different interesting creative methods of teaching such as assigning work in groups students learn from their peers easily. Peer can motivate their friends for working hard and developing positive attitude in them. Positive attitude is the key to success so if one succeeded in developing the positive attitude in students the conceptual clarity would be there automatically.

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